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ELASTIC EQUILIBRIUM OF A COMPOSITE PLANE CONTAINING AN ARBITRARILY ORIENTED THIN ELASTIC INCLUSION*

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The integral Mellin transform is used to study the stress-strain state of a piecewise homogeneous plane consisting of two bonded, isotropic half-planes, one of them containing an elastic, thin-walled inclusion of finite length. Solutions of the problems of stress concentration near an elastic inclusion in a homogeneous plane /1/ and the problem of a mathematical cut in a piecewise homogeneous plane /2/ are found to be particular cases of the results obtained in the present paper. However, unlike in /2/, the choice of the path around the pole in the process of estimating the integral equations obtained, is substantiated.

When the mass forces are absent, the solution of the plane problem of the theory of elasticity reduces to the problem of obtaining a general solution of the following differential equations /3/:

$$\nabla^4\varphi = 0, \quad \nabla^2\psi = 0, \quad \frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) = \nabla^2\varphi \tag{1}$$

In a polar $r\theta$-coordinate system the stresses $\{\tau_{r\theta}, \tau_{\theta\theta}, \tau_{rr}\}$ and displacements $\{u_r, u_\theta\}$ are expressed in terms of the functions φ and ψ by the relations

$$\sigma(r,\theta) = \tau_{r\theta} + i\tau_{\theta\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\varphi}{\partial\theta} + i\frac{\partial^2\varphi}{\partial r^2}\right), \quad \tau_{rr} = \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r}, \quad v(r,\theta) = \frac{\partial u_r}{\partial r} + i\frac{\partial u_\theta}{\partial r} = \tag{2}$$

$$\frac{1}{2\mu}\left\{-\frac{\partial^2\varphi}{\partial r^2} + \frac{i}{r^2}\frac{\partial\varphi}{\partial\theta} - \frac{i}{r}\frac{\partial^2\varphi}{\partial r\partial\theta} + \frac{(1+\varkappa)}{4}\left[\frac{\partial\varphi}{\partial\theta} + 2ir\frac{\partial\psi}{\partial r} + r\frac{\partial^2\psi}{\partial r\partial\theta} + ir^2\frac{\partial^2\psi}{\partial r^2}\right]\right\}$$

where $\varkappa = 3-4\nu$ for a plane deformation, $\varkappa = (3-\nu)/(1+\nu)$ for the generalized, plane stress state $\mu = E/[2(1+\nu)]$, E, and ν are the Young's modulus and the Poisson's ratio, respectively.

The solution of the problem dealing with the influence of an elastic thin-walled inclusion in the stress— strain state of a piecewise homogeneous plane under a given external load (Fig.1) can be regarded, within the limits of the linear theory of elasticity, as a superposition of the solutions of two problems. The problems are the first boundary value problem for a composite plane without an inclusion (we denote the corresponding quantities by the zero superscript), and the mixed boundary value problem for a piecewise homogeneous plane with a mathematical cut along the segment $[a, b]$ coinciding with the middle line of the inclusion (we denote the corresponding quantities by an asterisk)

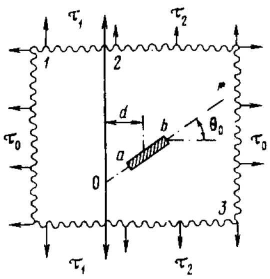

Fig.1

$$\begin{gathered}\sigma_1^*(r, \pi/2) = \sigma_2^*(r, \pi/2),\ v_1^*(r, \pi/2) = v_2^*(r, \pi/2) \\ \sigma_1^*(r, 3\pi/2) = \sigma_3^*(r, -\pi/2),\quad v_1^*(r, 3\pi/2) = v_3^*(r, \\ -\pi/2),\quad 0 < r < \infty\end{gathered} \tag{3}$$

$$\begin{gathered}\sigma_p^*(r, \theta_0\pm) = \sigma_p(r, \theta_0\pm) - \sigma_p^0(r, \theta_0) \\ v_p^*(r, \theta_0\pm) = v_p(r, \theta_0\pm) - v_p^0(r, \theta_0),\quad a < r < b\end{gathered} \tag{4}$$

The plus sign in the relations (4) corresponds to $p = 2$ and the minus sign to $p = 3$, and the indices 1,2 and 3 denote the quantities belonging to one of the three wedges

$$1)\ (\mu_1, \varkappa_1),\ \ {}^1/_2\,\pi < \theta < {}^3/_2\,\pi;\ 2)\ (\mu_2, \varkappa_2),\ \ \theta_0 < \theta < {}^1/_2\pi;\ 3)\ (\mu_2, \varkappa_2),\ -{}^1/_2\,\pi < \theta < \theta_0$$

of which the elastic body in question is composed.

The assumption that the inclusion is thin, enables us to assert that its presence is equivalent to the appearance of the stress and displacement discontinuities at the middle line of the interlayer

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$$\sigma_2(r,\theta_0+)-\sigma_3(r,\theta_0-)=\begin{cases} f_1(r)+if_2(r), & a\leqslant r\leqslant b \\ 0, & 0<r<a, \quad b<r<\infty \end{cases} \tag{5}$$
$$v_2(r,\theta_0+)-v_3(r,\theta_0-)=\begin{cases} f_3(r)+if_4(r), & a\leqslant r\leqslant b \\ 0, & 0<r<a, \quad b<r<\infty \end{cases}$$

and we have

$$\int_a^b f_j(r)\,dr = C^j \tag{6}$$

where C^j $(j=1,\ldots,4)$ denote known real constants /1/.

We define the Mellin transform of the function $f(r)$, defined and regular in the range $0<r<\infty$, and its inverse transform, as follows /4/:

$$F(s)=M[f,s]=\int_0^\infty f(r)\,r^{s-1}\,dr, \quad f(r)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} F(s)\,r^{-s}\,ds \tag{7}$$

Here the constant c is chosen from the condition of absolute integrability of $r^{c-1}f(r)$ on $(0,\infty)$. Applying the transform (7) to (1) and (2) we obtain, for the case of a wedge of an arbitrary angle θ,

$$M[r^2\sigma,\ s]=2i(s+1)[Ase^{is\theta}+B(s+1)e^{i(s+2)\theta}-\bar{B}e^{-i(s+2)\theta}] \tag{8}$$
$$M[r^2v,\ s]=(s+1)[Ase^{is\theta}+B(s+1)e^{i(s+2)\theta}+\varkappa\bar{B}e^{-i(s+2)\theta}]/\mu$$

where A and B are complex functions of the transform parameter s.

Let us also define

$$U_j(s)=M[r^2f_j(r),s]=\int_a^b f_j(r)\,r^{s+1}\,dr \tag{9}$$

Substituting (8) into the boundary conditions (3) and relations (5), with (9) taken into account, yields a system of six linear algebraic equations. Solving these equations we obtain $A_p(s)$, $B_p(s)$ $(p=1,2,3)$ in terms of $U_j(s)$ $(j=1,\ldots,4)$. Substituting the values of $A_p(s)$ and $B_p(s)$ into (8) and applying the transformation formula (7), we obtain from (4)

$$\frac{1+\varkappa_2}{\mu_2}\sigma_2^*(r,\theta_0+)=\frac{1+\varkappa_2}{\mu_2}\sigma_2^\circ(r)+\lim_{\theta\to\theta_0+0}R_1(r,\theta) \tag{10}$$

$$(1+\varkappa_2)\,v_2^*(r,\ \theta_0+)=(1+\varkappa_2)\,v_2^\circ(r)+\lim_{\theta\to\theta_0+0}R_2(r,\ \theta), \quad a\leqslant r\leqslant b$$

where

$$R_q(r,\theta)=\frac{1}{\pi}\int_a^b\sum_{j=1}^4 K_{qj}(r,r_0,\theta)f_j(r_0)\,dr_0, \quad K_{qj}(r,r_0,\theta)=\int_{c-i\infty}^{c+i\infty}\frac{H_{qj}(s,\theta)}{e^{-i\pi s}-e^{i\pi s}}\frac{r_0^{s+1}}{r^{s+2}}\,ds, \quad q=1,2;\ j=1,\ldots,4 \tag{11}$$

$$\begin{aligned}
&2\mu_2 iH_{11}=l_1^+ + l_2^+ - d_{12}^+ - d_3 + d_4 + l_3^+ - l_4^+ - d_{57}^+ - d_{68}^+\\
&2\mu_2 H_{12}=-l_1^- + l_2^- - d_{12}^- - d_3 - d_4 - l_3^- - l_4^- - d_{57}^- - d_{68}^+\\
&H_{13}=l_1^- - l_2^- - l_3^- + l_4^- + d_{57}^+ + d_{68}^+\\
&-iH_{14}=l_1^+ + l_2^+ + l_3^+ - l_4^+ + l_{57}^- - l_{68}^-\\
&4\mu_2 H_{21}=l_1^+ + l_2^+ - d_{12}^+ - d_3 + \varkappa_2 d_4 + l_3^+ + \varkappa_2 l_4^+ + d_{57}^- + \varkappa_2 d_{68}^-\\
&-4\mu_2 iH_{22}=l_1^- - l_2^- + d_{12}^- + d_3 - \varkappa_2 d_4 + l_3^- - \varkappa_2 l_4^- + d_{57}^+ - \varkappa_2 d_{68}^+\\
&2iH_{23}=-l_1^- + l_2^- - l_3^- + \varkappa_2 l_4^- - d_{57}^+ + \varkappa_2 d_{68}^+\\
&2H_{24}=l_1^+ + l_2^+ + l_3^+ + \varkappa_2 l_4^+ + d_{57}^- + \varkappa_2 d_{68}^-\\
&l_1^+ = m_1(s+1)(s+1\pm 1)e_{11}+[m_2+m_1(s+1)^2]e_{12}\\
&l_2^\pm=(s+1\pm 1)e_{13}, \quad l_3^\pm=m_1(s+1)(s+1\pm 1)e_{21}\\
&l_4^\pm=m_1(s+1\pm 1)/e_{21}, \ d_{12}^\pm=d_1\pm d_2, \quad d_{57}^\pm=d_5\pm d_7\\
&d_{68}=d_6\pm d_8, \quad d_1=m_1(1+\varkappa_2)(s+1)e_{11}\\
&d_2=(1+\varkappa_2)e_{13}, \quad d_3=m_1(1+\varkappa_2)(s+1)e_{21}\\
&d_4=m_1(1+\varkappa_2)/e_{21}, \quad d_5=m_1(s+1)^2e_{22}, \ d_6=m_1(s+1)e_{22}\\
&d_7=(s+1)e_{24}, \quad d_8=1/e_{24}, \quad e_{kp}(s,\theta,\theta_0)=e_k(s,\theta)e_p(s,\theta_0)\\
&(k,\ p=1,\ldots,4), \ e_1=e^{is\theta}\\
&e_2=e^{i(s+2)\theta}, \quad e_3=e^{-is(\theta+\pi)}, \quad e_4=e^{-i(s\theta+2\theta+s\pi)}\\
&m_1=(m-1)/(\varkappa_2 m-1), \quad m_2=(\varkappa_1-m\varkappa_2)/(\varkappa_1+m),\\
&m=\mu_1/\mu_2
\end{aligned}$$

We compute the integrals in (11) by integration along the real axis. To do this we must determine the constant c in a strip in which the integrand functions are regular. The integrand functions are analytic functions of the complex parameter s, except at the points $s_j = j$ ($j = 0, \pm 1, \pm 2, \ldots$) which are simple poles. Since $\tau_{ij} = o(r^{-\alpha})$, $\mathrm{Re}(\alpha) \geqslant 1$ as $r \to \infty$ and the residues of the integrand functions in (11) have the form $r^{-(s_j+2)}$, therefore the constant c should vary within the limits $-2 < c \leqslant -1$. Let us set $c = -1$ for simplicity. Then the substitution $s = -1 + iy$ reduces the integration in (11) to that along the real axis, i.e.

$$K_{qj}(r, r_0, \theta) = \frac{1}{r}\int_{-\infty}^{\infty} \frac{i\,dy}{e^{\pi y} - e^{-\pi y}}\left(\frac{r_0}{r}\right)^{iy} H_{qj}(-1 + iy, \theta)$$

$$\frac{1}{r}\int_{-\infty}^{\infty} \frac{i\,dy}{e^{\pi y} - e^{-\pi y}}\left(\frac{r_0}{r}\right)^{iy} H_{qj}(-1 + iy, \theta) = \tag{12}$$

$$\text{v. p.}\,\frac{1}{r}\int_{-\infty}^{\infty} \frac{i\,dy}{e^{\pi y} - e^{-\pi y}}\left(\frac{r_0}{r}\right)^{iy} H_{qj}(-1 + iy, \theta) \pm \pi i a_{qj}$$

$$a_{qj} = \operatorname*{Res}_{s=-1}\left[\frac{1}{e^{-i\pi s} - e^{i\pi s}}\,\frac{r_0^{s+1}}{r^{s+2}} H_{qj}(s, \theta)\right] = \frac{a_{qj}'}{r}, \quad q = 1, 2$$

$$j = 1, \ldots, 4$$

$$a_{11}' = \frac{i}{2\mu_2}\Delta_1, \quad a_{12}' = -\frac{1}{2\mu_2}\Delta_1, \quad a_{13}' = \Delta_2, \quad a_{34}' = i\Delta_2$$

$$a_{21}' = \frac{1}{4\mu_2}\Delta_3, \quad a_{22}' = \frac{i}{4\mu_2}\Delta_3, \quad a_{23}' = -\frac{i}{2}\Delta_1, \quad a_{24}' = \frac{1}{2}\Delta_1$$

$$\Delta_1 = m_2 + m_1\varkappa_2 - 1 + \varkappa_2, \quad \Delta_2 = m_2 - m_1 - 2, \quad \Delta_3 = m_2 - m_1\varkappa_2^2 + 2\varkappa_2$$

The plus or minus sign in (12) is chosen according to the direction in which the pole $s = -1$ is circumscribed, and the direction is determined as follows. We require the solution of the static problem posed to be a limiting case of the corresponding dynamic problem with the same boundary conditions (3), (4) when the spectral parameter or the inertial terms which we shall denote by ε^2, tend to zero ($\varepsilon \to 0$). When $\varepsilon \neq 0$ the denominator of the integrand expression in (11) assumes the form

$$\exp\left[-i\pi\sqrt{s^2 - \varepsilon^2}\right] - \exp\left[i\pi\sqrt{s^2 - \varepsilon^2}\right]$$

and $s_{j\varepsilon} = \pm\sqrt{s_j^2 + \varepsilon^2}$ will be its zeros. From this it follows that, as $\varepsilon \to 0$, the pole $s_{j\varepsilon}$ approaches the point $s = -1$ from the right. To preserve the continuous character of the solution we must ensure that the contour of integration does not intersect the path of this pole (Fig.2). The plus and minus signs in (12) should be taken for $r_0 > r$ and $r_0 < r$, respectively. Since

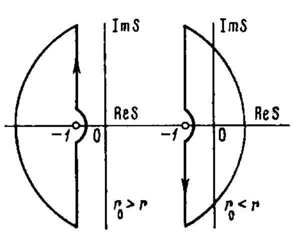

Fig.2

$$\frac{1}{r\ln(r_0/r)} = \frac{1}{r_0 - r}\left[1 + o\left(\frac{r_0}{r} - 1\right)\right], \quad \lim_{\varepsilon\to 0+}\int_0^{\infty} e^{-\varepsilon y}(\cos\rho y + i\sin\rho y)\,dy = \pi\delta(\rho) + \frac{i}{\rho}, \quad \left(\rho = \ln\frac{r_0}{r}, \quad \varepsilon = \theta - \theta_0\right)$$

where $\delta(\rho)$ is the delta function, the relations (10) and (12) yield expressions for the characteristics of the stress-strain state of the composite plane at the lower edge of the inclusion in the form

$$\tau_{r\theta}(r, \theta_0+) = \tau_{r\theta}^{\circ}(r) - l_1^+ G_1(r) + \frac{1}{2} f_1(r) + m_{12}^- t_2(r) - l_1^+ t_3(r) + l_1^+ Q_1(r) \tag{13}$$

$$\tau_{\theta\theta}(r, \theta_0+) = \tau_{\theta\theta}^{\circ}(r) + l_1^+ G_2(r) + \frac{1}{2} f_2(r) - m_{12}^- t_1(r) - l_1^+ t_4(r) + l_1^+ Q_2(r)$$

$$\frac{\partial u_r(r, \theta_0+)}{\partial r} = \frac{\partial u_r^{\circ}(r)}{\partial r} + \frac{G_3(r)}{1 + \varkappa_2} + \frac{1}{2} f_3(r) + l_2^+ t_2(r) + m_{21}^- t_3(r) + \frac{Q_4(r)}{1 + \varkappa_2}$$

$$\frac{\partial u_\theta(r, \theta_0+)}{\partial r} = \frac{\partial u_\theta^{\circ}(r)}{\partial r} + \frac{G_4(r)}{1 + \varkappa_2} + \frac{1}{2} f_4(r) + l_2^+ t_1(r) - m_{21}^- t_4(r) + \frac{Q_3(r)}{1 + \varkappa_2}$$

$$G_1(r)=\frac{1}{2\pi r}(a_{12}'C^2-a_{13}'C^3),\quad G_2(r)=\frac{1}{2\pi r}(a_{11}'C^1+a_{14}'C^4)$$
$$G_3(r)=\frac{1}{\pi r}(a_{21}'C^1+a_{24}'C^4),\quad G_4(r)=\frac{1}{\pi r}(a_{22}'C^2-a_{23}'C^3)$$
$$Q_i(r)=\frac{1}{\pi}\int_a^b\sum_{j=1}^4 k_{ij}(r,r_0)f_j(r_0)\,dr_0,\quad t_i(r)=\frac{1}{\pi}\int_a^b\frac{f_i(r_0)}{r_0-r}\,dr_0$$

where $k_{ij}(r, r_0)$ $(i, j = 1, \ldots, 4)$ denote the regular Fredholm kernels. The symbolism used to describe the remaining quantities follows, unless otherwise indicated, that of /1/.

Substituting the expressions (13) into the conditions of interaction between an elastic, thin-walled inclusion and the surrounding medium /1/ leads to a system of singular integral equations of the first kind

$$\begin{aligned}&t_1(r)+\lambda_{11}t_4(r)-\lambda_1I_1^{\cdot}(r)+\lambda_{12}Q_2(r)+\lambda_{13}Q_3(r)=F_1(r)\\&t_3(r)+\lambda_{21}t_2(r)-\lambda_2I_3^{\cdot}(r)-\lambda_{22}Q_1(r)+\lambda_{23}Q_4(r)=F_2(r)\\&t_4(r)+\lambda_{31}t_1(r)+\lambda_3I_1^{\cdot}(r)+\lambda_4I_4^{\cdot}(r)-Q_2(r)=F_3(r)\\&I_j^{\cdot}(r)=\int_a^r f_j(r_0)\,dr_0,\quad j=1,\ldots,4,\quad f_3(r)=-k_2f_2(r),\quad a\leqslant r\leqslant b\end{aligned}\tag{14}$$

where

$$\begin{aligned}&\Lambda_1F_1(r)=k_0N_a-\frac{\partial u_r^{\circ}(r)}{\partial r}-k_1\tau_{\theta\theta}^{c}(r)-k_1l_1^{+}G_2(r)-\frac{G_3(r)}{1+\varkappa_2}\\&\Lambda_2F_2(r)=\mu_0\left[\frac{\tau_{r\theta}^{\circ}}{\mu_0}-\frac{\partial u_\theta^{\circ}}{\partial r}+\frac{c_a}{2h}-\frac{l_1^{+}}{\mu_0}G_1(r)-\frac{G_4(r)}{1+\varkappa_2}\right]\\&l_1^{+}F_3(r)=\tau_{\theta\theta}^{\circ}+\frac{d_a}{2hk_0}-\frac{k_1}{k_0}N_a+l_1^{+}G_2(r)\\&\lambda_{12}=k_1l_1^{+}/\Lambda_1,\quad \lambda_{13}=1/[(1+\varkappa_2)\Lambda_1]\\&\lambda_{22}=l_1^{+}/\Lambda_2,\quad \lambda_{23}=\mu_0/[(1+\varkappa_2)\Lambda_2]\end{aligned}$$

The normal stresses N_a and displacements of the upper points of the face $s = a$ relative to the lower points c_a and d_a, are computed using the approximate formulas from /1/.

When the inclusion is perfectly rigid $(E_0 = \infty)$, the equations (14) imply that $f_3(r) = f_4(r) = 0$, and we obtain

$$t_1(r)-\frac{2\mu_2}{\varkappa_2}\frac{1}{\pi}\int_a^b\sum_{j=1}^2 k_{3j}(r,r_0)f_j(r_0)\,dr_0=-\frac{2\mu_2(1+\varkappa_2)}{\varkappa_2}\frac{\partial u_r^{\circ}(r)}{\partial r}\tag{15}$$

$$t_2(r)-\frac{2\mu_2}{\varkappa_2}\frac{1}{\pi}\int_a^b\sum_{j=1}^2 k_{4j}(r,r_0)f_j(r_0)\,dr_0=-\frac{2\mu_2(1+\varkappa_2)}{\varkappa_2}\frac{\partial u_\theta^{c}(r)}{\partial r},\qquad a\leqslant r\leqslant b$$

for determining the stress discontinuities. If on the other hand $E_0 = 0$, then (14) yield $f_1(r) = f_2(r) = 0$ and a system of equations describing the elastic equilibrium of the bounded half-planes made of different materials and containing a mathematical cut near the line of contact

$$t_3(r)+\frac{1}{\pi}\int_a^b\sum_{j=3}^4 k_{1j}(r,r_0)f_j(r_0)\,dr_0=l_1^{+}\tau_{r\theta}^{\circ}(r),\quad t_4(r)+\frac{1}{\pi}\int_a^b\sum_{j=3}^4 k_{2j}(r,r_0)f_j(r_0)\,dr_0=l_1^{+}\tau_{\theta\theta}^{\circ}(r),\qquad a\leqslant r\leqslant b\tag{16}$$

Since the index of the system (14) of singular integral equation $k = 1$, its solution must contain four, real arbitrary constants determined from the supplementary conditions (6). We seek the solution of the integral equations (14) with singular Cauchy type kernels in the form

$$f_j(r)=g_j(r)\,[(b-r)(r-a)]^{-1/2},\quad a\leqslant r\leqslant b,\quad j=1,\ldots,4$$

where the unknown functions $g_j(r)$ are bounded in a closed interval $[a, b]$. Using the method of orthogonal polynomials /5/, we obtain from (14) a system of linear algebraic equations for determining $g_j(r)$ at specified nodal points, the latter being the roots of the first order Chebyshev polynomials.

The normal and tangential component of the stress intensity coefficient are given by the formula

$$k_2(a)+ik_1(a)=\lim_{r\to a-0}\sqrt{2(r-a)}\,[\tau_{2r\theta}(r,\theta_0)+i\tau_{2\theta\theta}(r,\theta_0)]=\lim_{r\to a-0}\frac{2\mu_2}{(1+\varkappa_2)}\sqrt{2(r-a)}\,[f_1(r)+if_2(r)]$$

One case of elastic equilibrium investigated in detail is that of an aluminium-epoxide composite $(E_1/E_2 = 22.2)$ containing an elastic, thin-walled inclusion of arbitrary relative rigidity E_0/E_2, acted upon by a homogeneous stress field $\tau_{1r\theta}(r, 0) = \tau_{2r\theta}(r, 0) = 0$, $\tau_{1\theta\theta}(r, 0) = \tau_1$, $\tau_{2\theta\theta}(r, 0) = \tau_2$ at infinity. The inclusion of thickness $2h$ is situated on the segment

$(\theta=\theta_0,\ a\leqslant r\leqslant b)$. The distance separating the center of the inclusion from the line d joining the materials of the half-planes and the length $2a_0$ of the inclusion are defined as follows:

$$d=\frac{(a+b)}{2}\cos\theta_0,\quad 2a_0=b-a$$

The influence of the relative rigidity on the magnitude of the stress intensity coefficients was studied for various values of the angle θ_0 of orientation of the inclusion

$$k_p'(a)=k_p(a)\,/\,(\tau_2\sqrt{a_0}),\quad p=1,2$$

where $\tau_2\sqrt{a_0}$ is the stress intensity coefficient in an infinite plane with a crack of length $2a_0$.

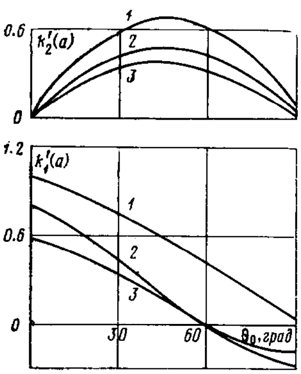

Fig.3

In the computations we assumed that $d=2a_0$, $a_0/h=10$, $\nu_0=\nu_1=0.3$, $\nu_2=0{,}35$. Fig.3 depicts the results obtained for the case of plane deformation. The curve *1*, *2* and *3* correspond to the values $E_0\,/\,E_2=0.01;\ 10;\ 100$ of relative rigidity of the inclusion. We note that the values obtained for $E_0\,/\,E_2=0.01$ differ from the corresponding results for a crack /2/ by not more than 3—4%.

Basically, the computation consisted of finding the kernels $k_{ij}(r,r_0)$, the latter representing semi-infinite integrals of the parameter of integration y. The following procedure was used to increase the accuracy of the computations. The interval of integration 0 to ∞ was divided into two parts by defining the point y_0. The functions $k_{ij}(r,r_0)$ were then computed from 0 to y_0 using the Filon quadrature formulas /6/, while over the remaining range y_0 to ∞ the integrand expressions were replaced by their asymptotic expansions with $y\to\infty$, the kernels $k_{ij}(r,r_0)$ obtained in closed form. Numerical analysis has shown that the contribution of these last terms in $k_{ij}(r,r_0)$ becomes significant for an inclusion near $(d\leqslant 2)$ the line along which the materials are bonded. Neglecting these contributions leads to erroneous results even at large (or the order of 200-400) values of y_0.

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